

# Non-Linear Finite Element Analysis Support Testing for Elastomer Parts

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## ABSTRACT

Computer-Aided Engineering (CAE) refers to the use of computers to perform design calculations for determining an optimum shape and size for a variety of engineering applications. This modern concept of engineering management has led to important advances in the design and production of components used in aerospace, automotive, electronics and other industries throughout the world.

CAE enables an engineer to test design ideas by simulating the function of the part on the computer. Finite Element Analysis (FEA) is one of these computer simulation techniques which is most accurate, versatile and comprehensive technique for solving complex design problems. FEA permits the analysis of these complex structures without the necessity of developing and applying complex equations.

FEA program for non-linear stress analysis of elastomers is performed by utilizing two material models:

- \* Mooney-Rivlin Model
- \* Ogden Model

The Mooney-Rivlin model is the most widely used model for elastomer analysis. The basic problem facing the design engineer is how to obtain the material coefficients needed to use these two models in FEA. As expected, the effectiveness of design analysis is directly related to the quality of the material input material coefficients.

Akron Rubber Development Laboratory, Inc. (ARDL) has developed a reliable history of standard procedures for determination of these coefficients from experimental test data. This paper will discuss various testing techniques used for developing elastomer material constants. Also, the intent of this paper is to show how aging or service conditions can be incorporated to obtain material coefficients for elastomer parts.

## INTRODUCTION

Computer Aided Engineering techniques, such as Finite Element Analysis, are increasingly used in the design of engineering rubber parts and components. These techniques are capable of solving complex structural problems which are not possible to solve by classical techniques. FEA also provides a way to achieve the desired force deflection characteristics and predict the geometry of the elastomeric part in use. FEA is a good tool for Failure Analysis, Failure Mode and Effect Analysis (FMEA). However, the accuracy and usefulness of solutions derived from the FEA depends on the accuracy of input material properties.

Several constitutive theories based on strain energy density functions have been developed for polymer elastomeric materials. These theories can be used very effectively with FEA to analyze and design elastomer parts which undergo high non-linear viscoelastic deformation. Unfortunately, more complicated constitutive equations that have improved accuracy also use more terms or constants in the models. These increased number of material constants require more lab testing to develop the data necessary to give good "curve fit" over a practical useful service stress-strain range.

Also, due to the complexities of the mathematical equations and the lack of standard general guidelines for characterizing the rubber material, it is not possible for a design engineer to make use of these constitutive equations. In the present paper, the test methods for developing the material constants for Mooney-Rivlin and Ogden Material Model is described. Also, the purpose of this paper is to show how we can incorporate aging or service conditions for evaluating material constants.

## THEORETICAL CONSIDERATION

A major factor in using Finite Element Analysis to model the behavior of an elastomer in an engineering component is the use of a mathematical model that represents the actual behavior. It is beyond the scope of this research paper to fully explain all the available mathematical models. However, it is appropriate to comment on the major models due to the large effect of the mathematical models on FEA.

### A. Mooney-Rivlin Material Model

The Mooney-Rivlin Material Model is the most widely used constitutive equation for Non-linear Finite Element Analysis modeling. The Mooney-Rivlin model states that elastic energy of an unstressed rubber material, isotropic and incompressible material, can be represented in terms of a strain-energy function  $W$ .

$$W = \sum_{ijk=0}^n C_{ijk} (I_1 - 3)^j (I_2 - 3)^j (I_3 - 1)^k \quad (1)$$

Where  $I_1$ ,  $I_2$  and  $I_3$  are the three invariants of the Green deformation tensor and are given by the following.

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 &= \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_1^2 \lambda_3^2 \\ I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 \end{aligned}$$

Where  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are like principle extension ratios,

extension ratio  $(\lambda) = \frac{\text{Final Length}}{\text{Original Length}} = 1 + \text{Strain } (\epsilon)$

Where strain  $(\epsilon) = \frac{\text{Current Length} - \text{Original Length}}{\text{Original Length}}$

As a result of the incompressibility condition ( $V = \text{Constant}$ )  $I_3 = 1$ , then the storable elastic energy of the network is only a function of  $I_1$  and  $I_2$ .

Equation (1) reduces to

$$W = \sum_{ij=0}^n C_{ij} (I_1 - 3)^j (I_2 - 3)^j \quad (2)$$

### 1. Neo-Hookean Material Model

Taking only the first terms of equation (2)

$$W = C_1 (I_1 - 3)$$

In the case of uniaxial tension:

$$t_1 = 2C_1(\lambda - \lambda^{-2}) \quad (3)$$

Where  $t_1$  = True stress (ratio of force to the current area)

$$\sigma_1 = 2C_1(1 - \lambda^{-3}) \quad (4)$$

Where  $\sigma_1$  = Engineering stress (ratio of force to the original area)

### 2. Mooney-Rivlin "Two Constant" Material Model

Taking only the first and second terms of equation (2)

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (5)$$

In the case of uniaxial tension: ( $\lambda_1 = \lambda$ ,  $\lambda_2 = \lambda_3 = \lambda^{-1/2}$ )

$$t_1 = 2(C_{10} + \frac{C_{01}}{\lambda})(\lambda^2 - \frac{1}{\lambda}) \quad (6)$$

$$\sigma_1 = 2(C_{10} + \frac{C_{01}}{\lambda})(\lambda - \frac{1}{\lambda^2}) \quad (7)$$

For most polymers  $0 < C_{01} < 0.2 C_{10}$ ; and small strain:

$$\sigma_1 = 6(C_{01} + C_{10}) \epsilon$$

In the case of pure shear:

$$(\lambda_1 = 1, \lambda_2 = \lambda, \lambda_3 = \frac{1}{\lambda^2})$$

$$t_1 = 2(C_{10} + C_{01})(\lambda^2 - \frac{1}{\lambda^2})$$

$$\sigma_1 = 2(C_{10} + C_{01})(\lambda - \frac{1}{\lambda^3})$$

For small strain:

$$\sigma_1 = 8(C_{10} + C_{01}) \epsilon$$

In the case of biaxial extension: ( $\lambda_1 = \lambda$ ,  $\lambda_2 = \lambda$ )

$$t_1 = 2(C_{10} + \lambda^2 C_{01})(\lambda^2 - \frac{1}{\lambda^4}) \quad (10)$$

$$\sigma_1 = 2(C_{10} + \lambda^2 C_{01})(\lambda - \frac{1}{\lambda^3}) \quad (11)$$

### 3. "Five-Constant" Strain Energy Model

Five-constant was derived by James, Green and Simpson based on 3rd order deformation theory and the 2nd order invariant theory of the Mooney-Rivlin equation by setting  $C_{12} = C_{21} = C_{03} = 0$

• 3rd Order Deformation Theory: ( $C_{02} = 0$  and uniaxial tension case)

$$\sigma_1 = 2(\lambda_1 - \lambda_1^{-2})\{C_{10}, C_{01}, C_{11}, C_{20}, C_{30}\} \quad (12)$$

This strain-energy function is incorporated into a number of finite element analysis codes.

• 2nd Order Invariant Theory: ( $C_{30} = 0$  and uniaxial tension case)

$$\sigma_1 = 2(\lambda_1 - \lambda_1^{-2})\{C_{10}, C_{01}, C_{11}, C_{20}, C_{02}\} \quad (13)$$

Higher order strain-energy functions has better fit to experimental data. However, it is found that prediction beyond the range of input data may bring serious error in the modeling.

### B. Ogden Material Model

The Ogden Material Model defines the strain-energy density as a function which can be considered as separate functions in major principle stretches.

For a simple tension, Engineering Stress ( $\sigma$ ) can be expressed as

$$\sigma_1 = \sum_j C_j [\lambda^{b_j-1} - \lambda^{-(1-0.5b_j)}] \quad (14)$$

For a simple tension, the Ogden formulation can be expressed as,

$$\sigma = \sum_j C_j [\lambda^{b_j-1} - \lambda^{-(1-0.5b_j)}] \quad (15)$$

Where  $\sigma$  is the engineering stress (force per unit original area) and  $\lambda$  is the uniaxial stretch ( $1 + dL/L_{\text{original}}$ ).

For a pure-shear case, the Ogden material model can be expressed as,

$$\sigma = \sum_j C_j [\lambda^{b_j-1} - \lambda^{-(1+b_j)}] \quad (16)$$

For an equi-biaxial tension, the Ogden material model can be expressed as,

$$\sigma = \sum_j C_j [\lambda^{b_j-1} - \lambda^{-(1+2b_j)}] \quad (17)$$

Any or all of the above three equations can be used to derive the Ogden coefficients  $C_j$  and  $b_j$ . A more detailed discussion of Ogden material model is given in reference 3.

The number of coefficients ( $C_1, C_2, C_3, \dots$ ) and ( $b_1, b_2, b_3, \dots$ ) needed fit the cure depends on the amount of accuracy desired by the user. Generally, it is observed that three sets of coefficients are sufficient to fit the data for most elastomers.

All the major FEA codes like ANSYS, ABAQUS, MSC/NASTRAN, COSMOS/M and MARC need Mooney-Rivlin constants or Ogden constants as inputs to run a non-linear analysis. For a rubber-like material, homogeneous deformation modes suffice to determine the material constants. The FEA codes accept data from the following deformation modes:

- Uniaxial tension (Figure 1)
- Uniaxial compression (Figure 2)
- Planar tension (Figure 3)

Note: The wide planar shear sample is physically constrained to not pull-in at the edges by the clamps, so the sample will thin out but not draw-in during the elongation. This mode of testing will place the specimen in a pure shear mode up to 100% elongation.

- Equi-biaxial tension (Figure 5)
- Volumetric compression (Figure 6)

These modes are shown in Figure 6. The most commonly performed experiments are uniaxial tension, uniaxial compression, planar tension and equi-biaxial tension. Combination of data from these four types of tests will result in good characterization of the behavior of the elastomer. The superposition of a tensile or compressive hydrostatic stress on a loaded, fully incompressible elastic body results in different stresses, but does not change the deformation. Figure 7 shows that different loading conditions are equivalent in their deformations, and therefore are equivalent in tests.

- Uniaxial tension = Equi-biaxial compression
- Uniaxial compression = Equi-biaxial tension (It maybe difficult this condition due to the experimental difficulty in performing a "true" uniaxial compression. Therefore, it is important to perform the equi-biaxial test for evaluating elastomer parts in compression)
- Planar tension = Planar compression
- The tensile and compression cases of the uniaxial and equi-biaxial modes are independent from each other. Therefore, uniaxial tension and uniaxial compression provide independent data.

The following procedure summarizes the steps need to calculate the material coefficients:

1. Run uniaxial tension, uniaxial compression, planar shear and biaxial tension tests for the test material. Perform the tests in slow speed to achieve a quasi-static condition.
2. Calculate the stress-strain data.
3. For Mooney-Rivlin 2-point model, perform a linear regression analysis on the experimental stress-strain data. To determine coefficients for 5-point model and Ogden model, perform a non-linear regression analysis on the experimental stress-strain data. The number of coefficients needed to fit the experimental curve depends on the amount of accuracy desired by the user.

## EXPERIMENTAL

The elastomer material evaluated were Silicone (PVMQ), Fluoroelastomer (FKM), and Hydrogenated Nitrile Rubber (HNBR). These rubber formulations were specifically compounded for gasket applications. The formulations are specified in Tables 1-3.

Table 1  
Hydrogenated Nitrile Rubber (HNBR)

Ingredient	phr
Zetpol 2010 <sup>a</sup>	100.0
N-774	45.0
PlastHall TOTM	5.0
Ladox 911C (ZnO)	5.0
Naugard 445	1.5
Vanox ZMTI	1.0
Dynamar PPA-790	1.0
Saret SR-517	8.0
Vul-Cup 40 KE	8.0

<sup>a</sup> Zeon Chemicals

Table 2  
Fluoroelastomer (FKM)

Ingredient	phr
FE 5640Q <sup>a</sup>	100.0
MT Black (N990)	30.0
MgO (Malite D)	3.0
Ca(OH) <sub>2</sub>	6.0

<sup>a</sup> 3M Specialty Fluoropolymers Department

Table 3  
Silicone (PVMQ)

Ingredients
Dow Corning 24092-V <sup>a</sup> third Generation Silicone Catalyzed with STI V

<sup>a</sup>Dow Corning STI

All three compounds were aged in three different fluids for 168 Hrs. at 150°C to see the change in stress-strain data. the three fluids were IRM-903, 5W-30 Engine oil and Automatic Transmission Fluid. All measurements were conducted at room temperature.

Measurements were made in uniaxial tension (Figure 1), uniaxial compression (Figure 2), planar tension (Figure 3), equi-biaxial (Figure 4), and volumetric compression (Figure 5).

The uniaxial tension and planar shear tests were performed using a Monsanto T-2000 screw type machine with a Monsanto E5042 laser Extensometer. The use of a laser extensometer to measure strain, rather than crosshead displacement, eliminates measurement error from slippage at the grips. The uniaxial compression test was performed on a MTS 831.20 Elastomer Test

System, and was performed on a cylindrical test specimens lubricated with silicone oil. The equi-biaxial tension was performed on a Iwamoto Biaxial Stretcher. All test were performed at room temperature at a strain rate of 0.2 inches per minute.

## RESULTS AND DISCUSSION

A comparison of the relative fit of linear model, Mooney-Rivlin models and Ogden model for tensile behavior of silicone rubber is shown in Figure 8. It is clear that linear equation follows the elastomeric behavior up to 10% strain level. The Mooney-Rivlin equations are a good fit up to 125% strain level. Ogden model showed the best fit with the measured tensile data over the full extension range.

The pure shear behavior of a planar specimen is compared I figure 9. The linear equation showed good fit at low strains. Mooney-Rivlin and Ogden models showed better fit with the measured engineering shear stress values over the full strain range.

The compression behavior of aged silicone rubber is compared using a equi-biaxial test shown in figure 10. The linear equation shows an acceptable fit only for very low strain values. Mooney-Rivlin equation yield a very good fit up to 40% strain. But, the Ogden model showed the best fit over the full strain range.

The compression behavior of unaged and aged silicone rubber is shown in Figure 1. This plot illustrates the importance of aging studies for modeling elastomer components used in oil environments. Figure 12 shows the compression behavior of FKM in 5W-30 Engine oil and Figure 13 shows the compression-deflection behavior of HNBR in Automatic Transmission Fluid.

## CONCLUSION

The test methodology for developing the material constants for Mooney-Rivlin and Ogden material models is shown from the viewpoint of a design/test engineer. Also demonstrated is how various aging techniques can be beneficially applied for obtaining material data which can be used for modeling elastomer parts in service.

Ogden model shows the best experimental data strain level up to 100%. Both linear an non-linear showed reasonable correlation at low strains (up to 10%). Standardizing test parameters and test speed is very important for obtaining reproducible data. Before doing a complete FEA on the part, it maybe necessary to verify the selected model on a simple geometry such as o-ring or button through the preprocessor. Incorporating fluid aging studies in the FEA support testing is important for obtaining meaniful material coefficient.

## ACKNOWLEDGMENTS

We gratefully acknowledge Mr. Robert Samples, ARDL's Chief Executive Officer for his support and encouragement given us during the preparation of this work.

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# Uniaxial Tension

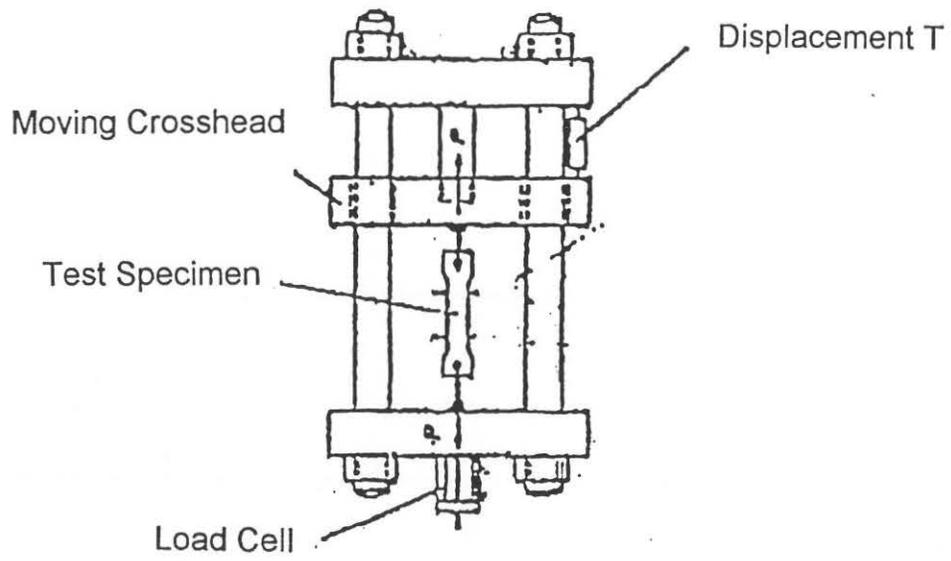


Figure 1

- Measurement Temperature: Room Temperature
- Specimen Size: ASTM Die "C" Dumbbell
- Test Rate: 0.2 in. per Minute

# Uniaxial Compression

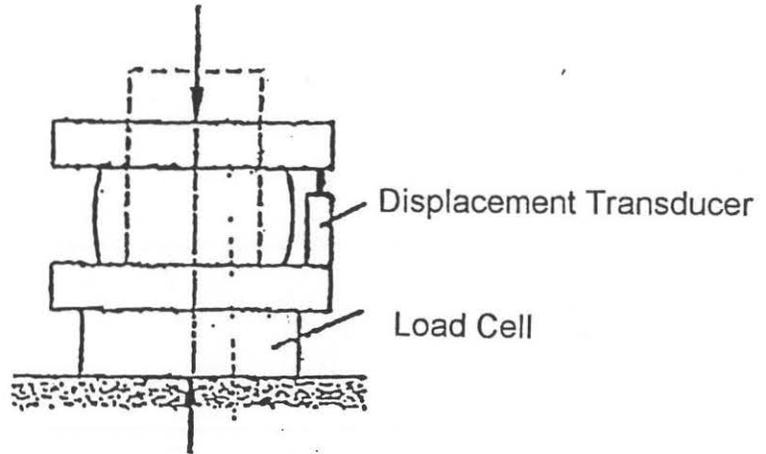


Figure 2

- Measurement Temperature: Room Temperature
- Specimen Size: ASTM Compression Set Button
- Test Rate: 0.2 in. per Minute

## Planar Shear

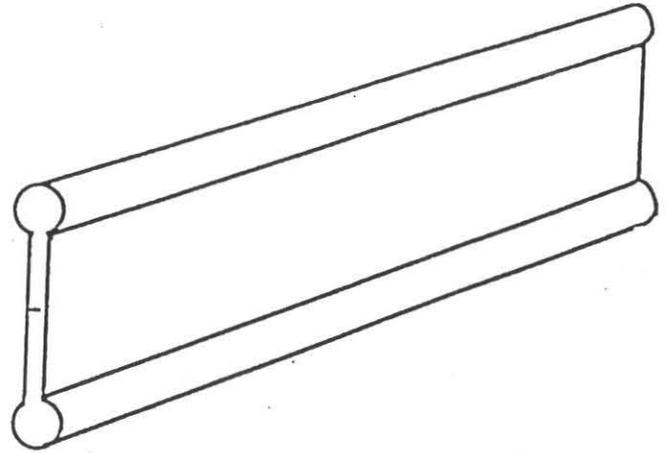
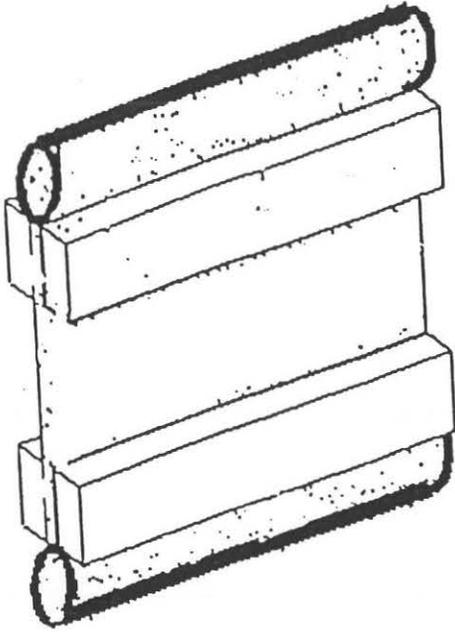


Figure 3

- Measurement Temperature: Room Temperature
- Specimen Size: 3.00 in. x 0.5 in. x 0.06 in. thick
- Test Rate: 0.2 in. per Minute

## Equi-Biaxial Tension

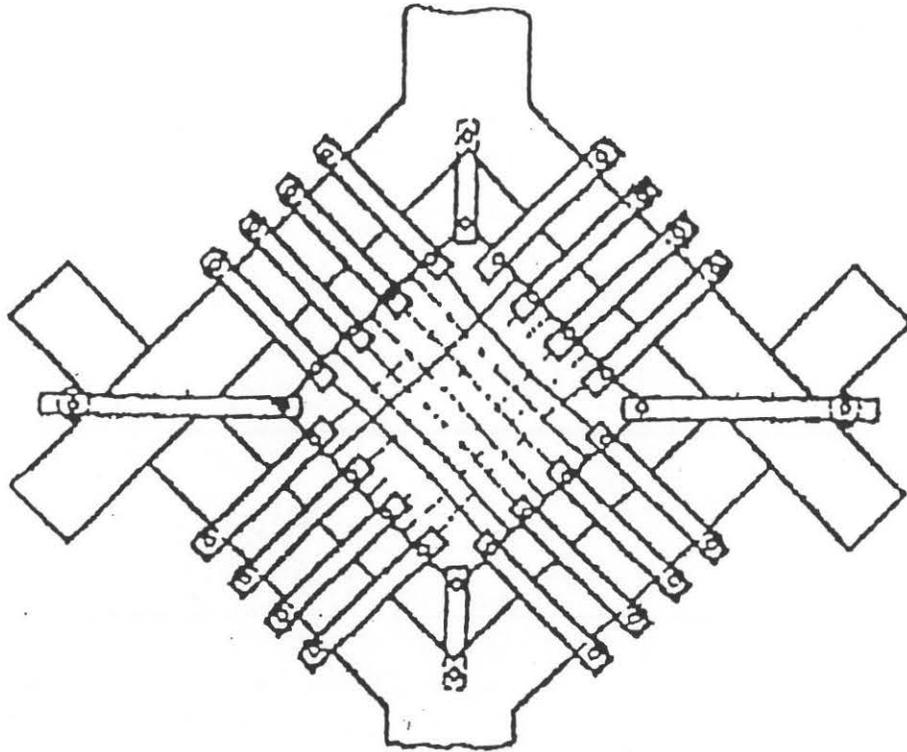


Figure 4

- Measurement Temperature: Room Temperature
- Specimen Size: 5.0 in. x 5.0 in. x 0.05in. thick
- Test Rate: 0.2 in. per Minute

## Volumetric Compression

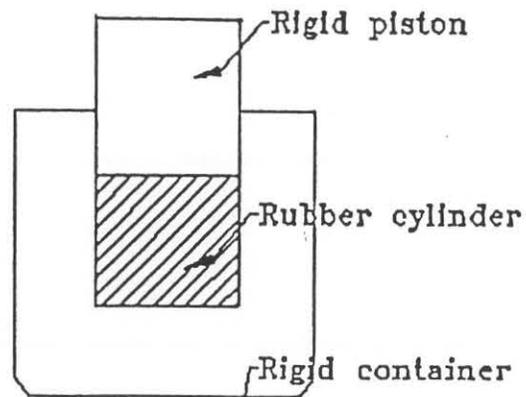
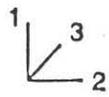


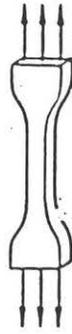
Figure 5

- Measurement Temperature: Room Temperature
- Specimen Size: 0.7 in. diameter x 1.0 in. thickness
- Procedure: Compression inside rigid container;  $E = 3 \times K(1 - (2 \times \nu))$   
E = Elastic Modulus; K = Bulk Modulus;  $\nu$  = Poisson's Ratio
- Test Rate: 0.2 in. per Minute

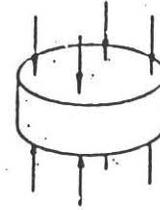
UNIAXIAL TEST DATA



TENSION

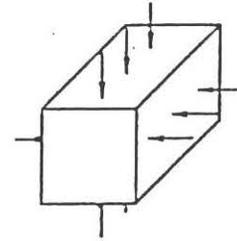
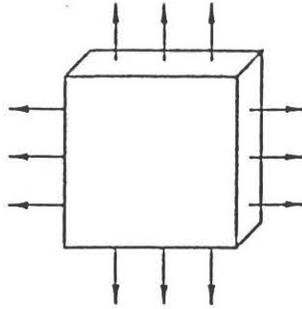
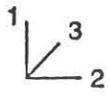


COMPRESSION



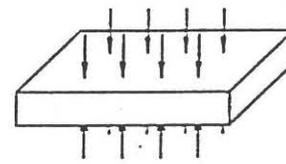
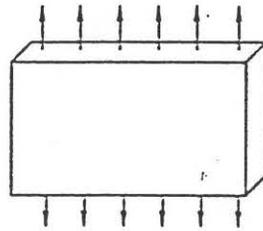
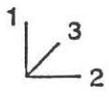
$$\lambda_1 = \lambda_U = 1 + \epsilon_U, \lambda_2 = \lambda_3 = 1/\sqrt{\lambda_U}$$

BIAXIAL TEST DATA



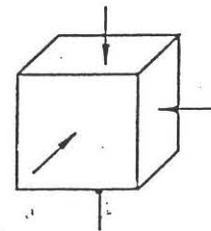
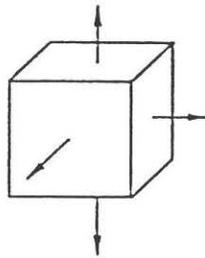
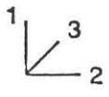
$$\lambda_1 = \lambda_2 = \lambda_B = 1 + \epsilon_B, \lambda_3 = 1/\lambda_B^2$$

PLANAR TEST DATA



$$\lambda_1 = \lambda_S = 1 + \epsilon_S, \lambda_2 = 1, \lambda_3 = 1/\lambda_S$$

VOLUMETRIC TEST DATA



$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_V, \frac{V}{V_0} = \lambda_V^3$$

Figure 6

Schematic illustrations of deformation modes

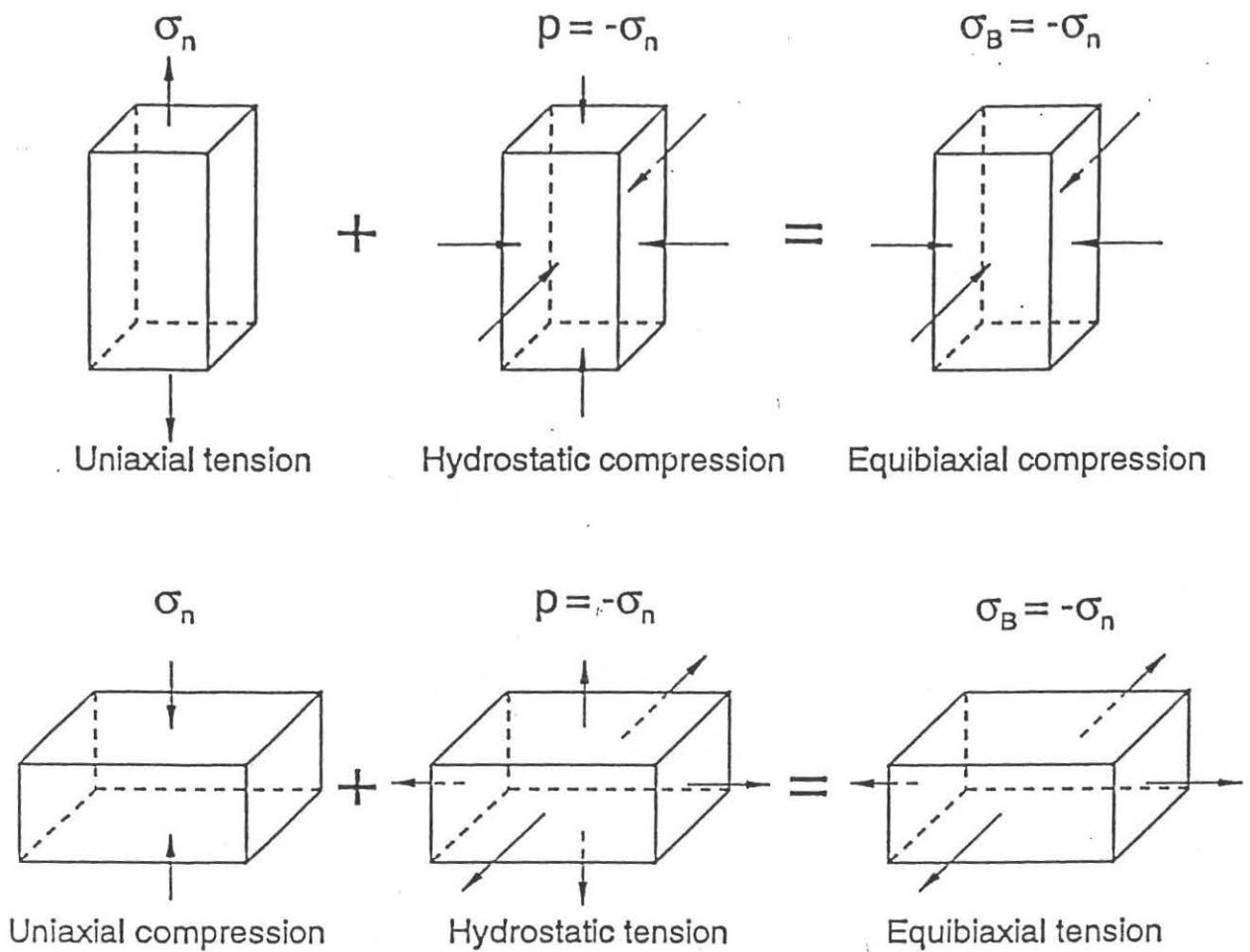


Figure 7

Schematic illustrations of equivalent deformation modes through superposition of hydrostatic stress. The stresses ( $\sigma_i$ ) shown in the figure are true stresses.

Figure 8  
Uniaxial Tension  
Silicone

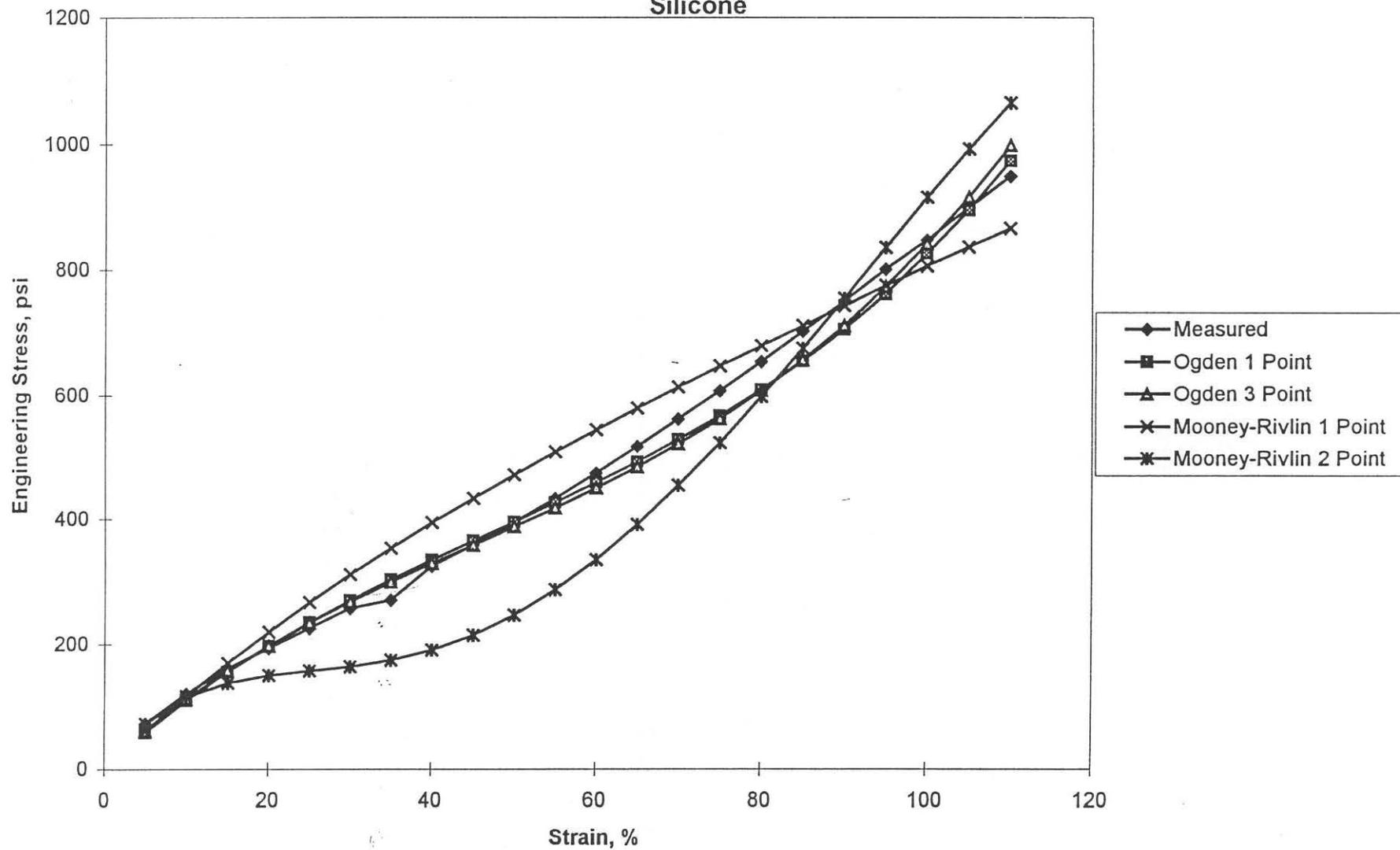


Figure 9  
Planar (Pure) Shear  
Silicone

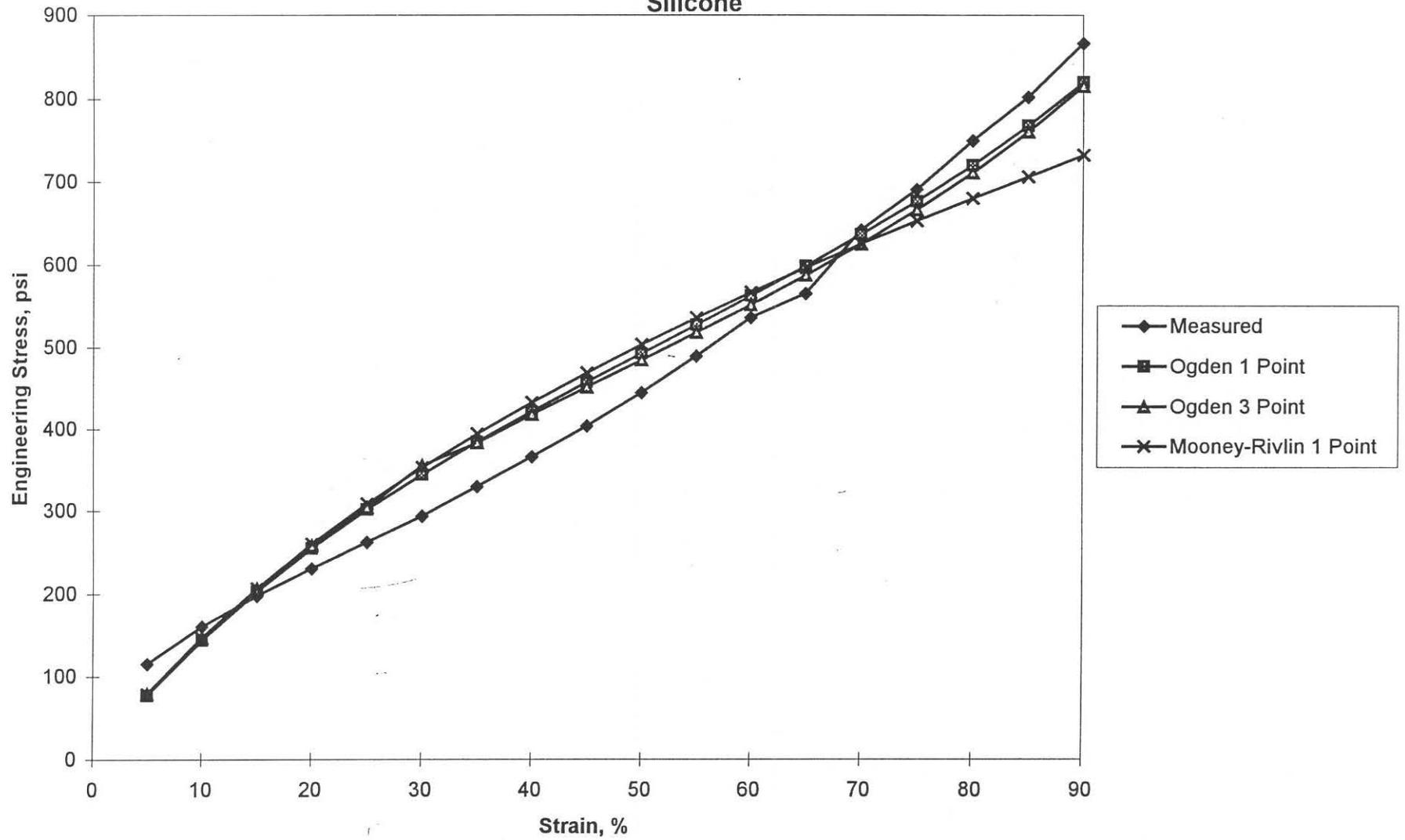


Figure 10  
Equi-Biaxial Tension  
Silicone

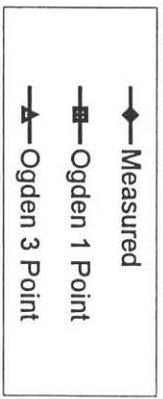
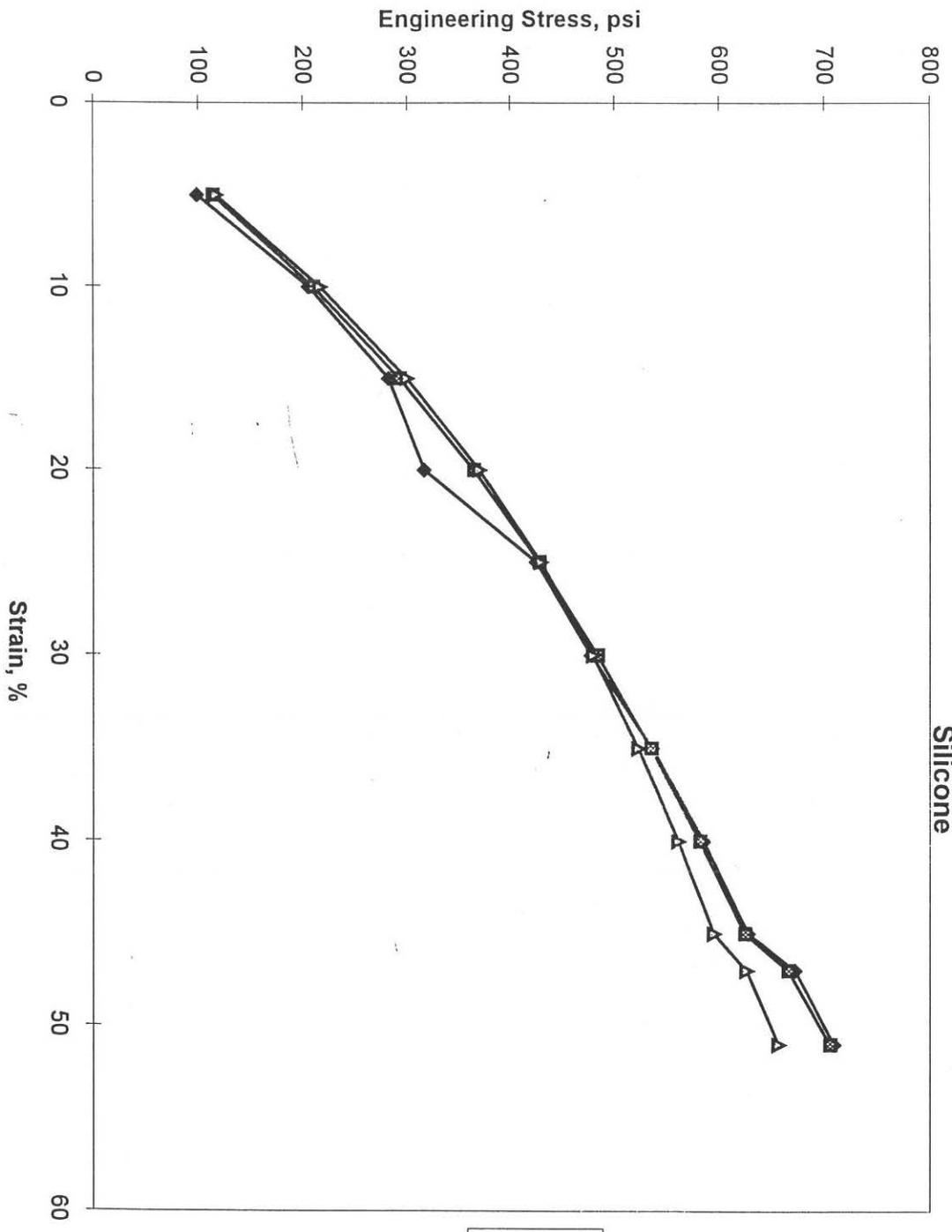


Figure 11  
Equi-Biaxial Tension  
Silicone; Aged in IRM 903 for 168 hrs.

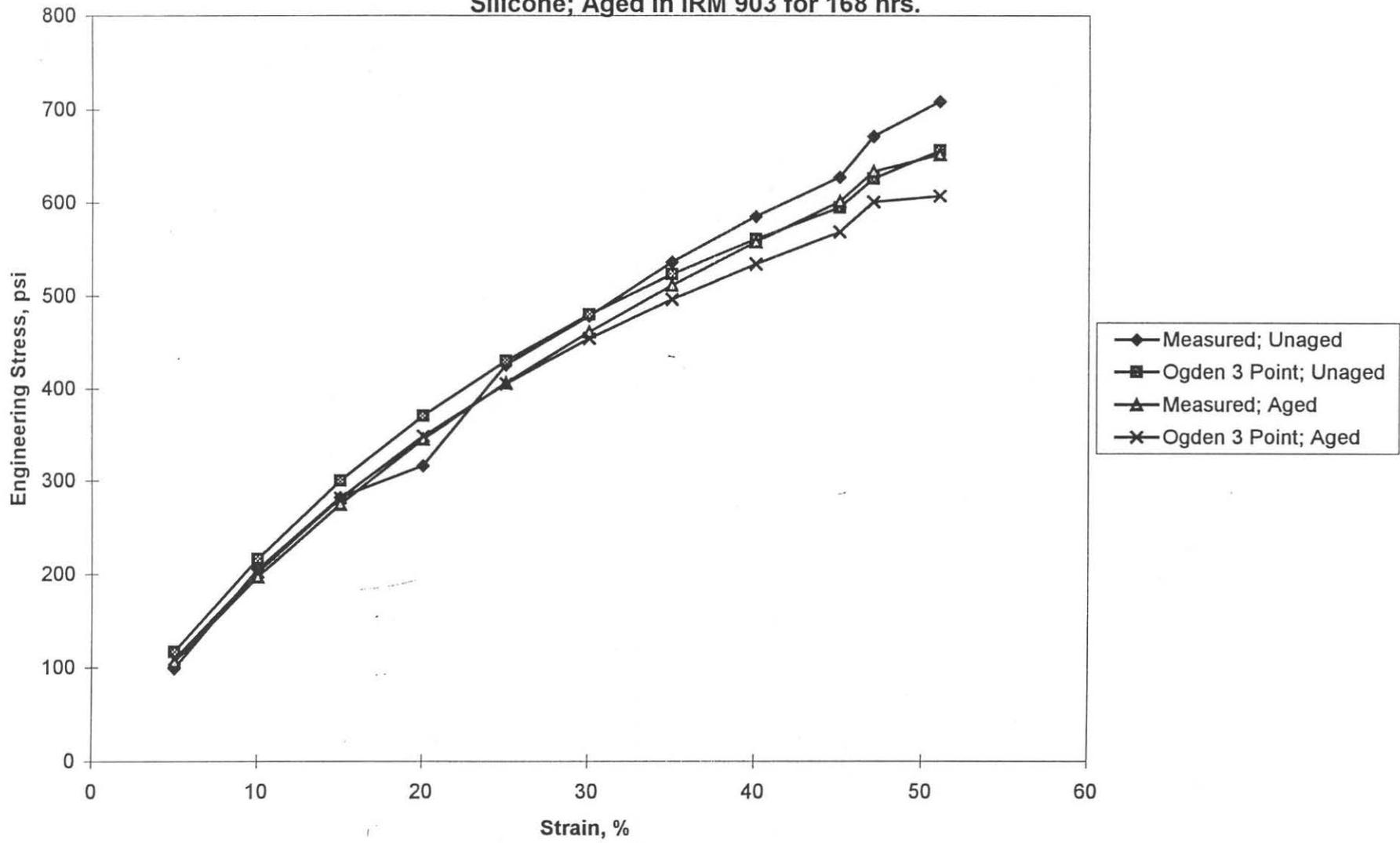


Figure 12  
Equi-Biaxial Tension  
FKM; Aged in 5W-30 Engine Oil for 168 hrs.

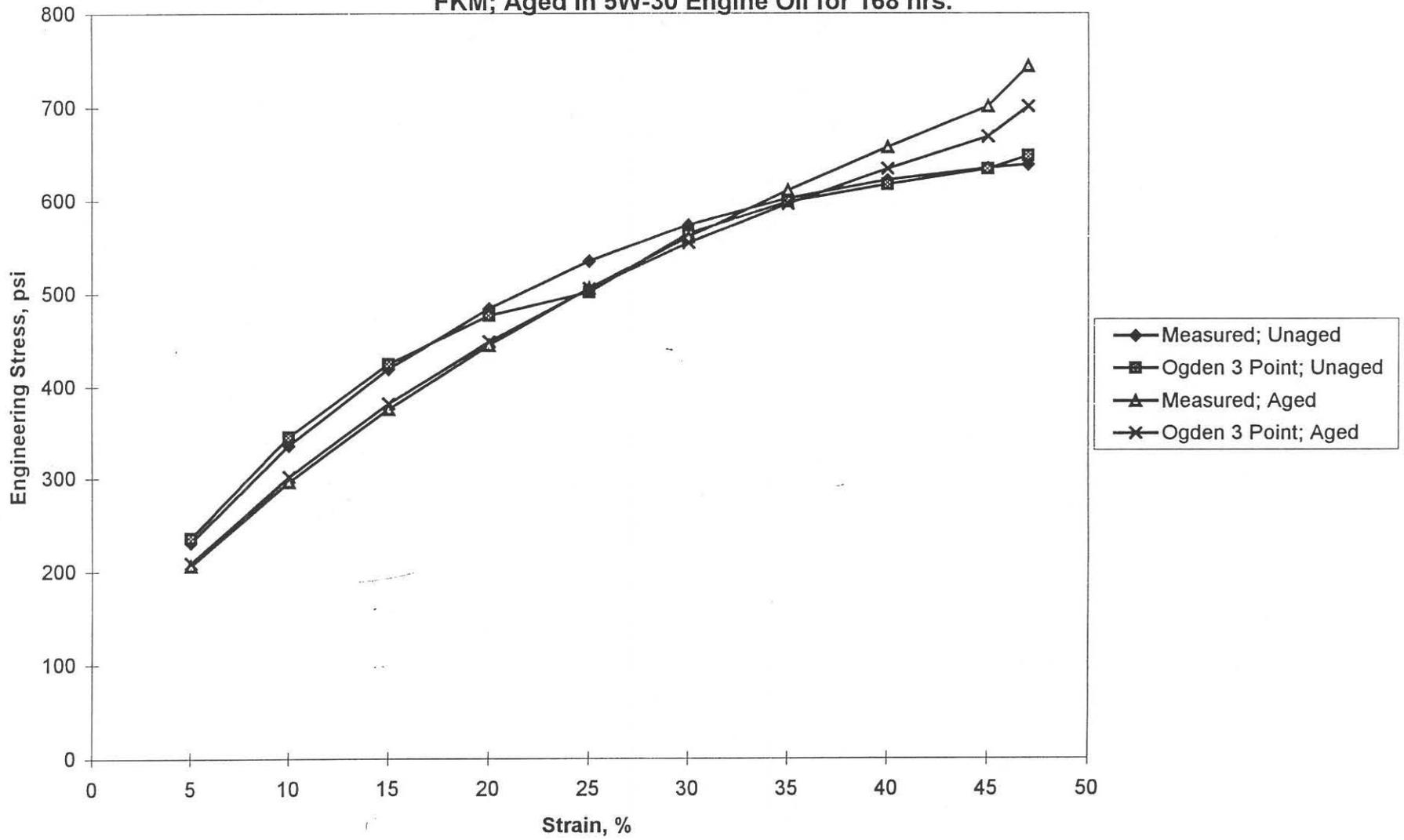


Figure 13  
Equi-Biaxial Tension

HNBR; Aged in Automatic Transmission Fluid for 168 hrs.

